

THE ANALYSIS OF THE ACOUSTIC WAVE VELOCITY ON THE TARGET LOCALIZATION ACCURACY

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Abstract

The accuracy of target localization using acoustic methods depends on atmospheric conditions. The acoustic wave velocity varies with fluctuation of atmospheric temperature, pressure and humidity. Therefore, a mathematical model for the description of acoustic wave velocity on target localization accuracy has been described. The results are presented and commented upon.

Key words: Target; Location; Acoustic wave; Wave velocity; Atmosphere; Humidity; Temperature.

Introduction

The enemy's target detection using acoustic methods is closely connected with the problems of the transmission channel – the atmosphere in this case. The information about the target can be widely improved by solving the problems connected with the transmission channel behaviour and target acoustic characteristic. The description of the acoustic wave propagation is usually linked to the neglecting of the atmosphere (transmission channel) characteristics. Alternatively the resultant data are only partially corrected.

Spatial atmospheric influence on acoustic wave propagation

The attenuation of the acoustic wave in the atmosphere depends on air composition with the significant influence of water vapour concentration. According to ISO 2533 the standard atmosphere, defined as a clean and dry air at sea level, consists of 78.084% of nitrogen, 20.9476% of oxygen and 0.0314% of carbon dioxide. The remaining percentage, e.g. 0.937% of the dry air pertains to trace elements that have no effect on acoustic wave attenuation. The said air composition is assumed up to 50 km above sea level for default calculation of the atmospheric attenuation. However the water vapour concentration varies in a wide spread. The water vapour concentration 10km above sea level can be 100 times lower than at sea level.

The increase of the acoustic pressure amplitude of the harmonic tone propagating through the atmosphere is described as an exponential increase characteristic. Thus for the current acoustic pressure amplitude p_t at the distance s the following is valid [6]:

$$p_t = p_i \cdot \exp(-0,1151 \cdot \alpha \cdot s) \quad (1)$$

Where p_i is an initial acoustic pressure amplitude, α is an attenuation coefficient and $0.1151 = 1/[10 \log(e^2)]$.

The attenuation of the atmosphere $\delta L_t(f)$ [dB] for harmonic tone with frequency f is than defined [6]:

$$\delta L_t(f) = 10 \cdot \log(p_i^2 / p_t^2) = \alpha \cdot s \quad (2)$$

In agreement with Avogadro's law, the molar concentration of water vapour is equal to the ratio of partial water vapour pressure to atmospheric pressure. The water vapour molar concentration ranges from 0.2% up to 2% for normal conditions at sea level. However it increases under the 0.01% for the high over 10km above the sea level.

The atmospheric attenuation, according to [6], is a function of two frequencies of relaxation: the oxygen frequency of relaxation f_{ro} and the nitrogen frequency of relaxation f_{rn} . The values of said frequencies are determined by the following equations:

$$f_{ro} = \frac{p_a}{p_r} \left(24 + 4,04 \cdot 10^4 h \frac{0,02 + h}{0,391 + h} \right) \quad (3)$$

$$f_{rN} = \frac{p_a}{p_r} \left(\frac{T}{T_0} \right)^{-1/2} \left\{ 9 + 280.h.\exp \left[-4,17. \left(\left(\frac{T}{T_0} \right)^{-1/3} \right) \right] \right\}. \quad (4)$$

The coefficient of attenuation caused by the atmosphere α [dB] is expressed by:

$$\alpha = 8,686.f^2 \left\{ \left[1,84.10^{-11} \left(\frac{p_a}{p_r} \right)^{-1} \left(\frac{T}{T_0} \right)^{1/2} \right] + \left(\frac{T}{T_0} \right)^{-5/2} \left[0,01275. \left(f_{r0} + \frac{f^2}{f_{r0}} \right)^{-1} \exp \left(\frac{-2239,1}{T} \right) + 0,1068. \left(f_{rN} + \frac{f^2}{f_{rN}} \right)^{-1} \exp \left(\frac{-3352,0}{T} \right) \right] \right\}. \quad (5)$$

Equations [3] and [5] are sufficient for the determination of the coefficient of attenuation of harmonic tones propagating through the atmosphere, the temperature of which is T; In fact the atmosphere is an acoustic atmospheric filter.

Generally, the dependence of the speed of sound c [m s⁻¹] on air temperature t [°] can be expressed as [3]:

$$c_1(t) = 331 \sqrt{1 + \frac{t}{273}}, \quad (6)$$

or

$$c_2(T) = 343,2 * \left(\frac{T}{T_0} \right)^{1/2}, \quad (7)$$

where $T_0 = 293.15K$. Another method for the speed of sound calculation is [2];

$$c_3 = 331,57 + 0,607t \quad (8)$$

The expressions [6,7,8] neglect the minor effect of water vapour on the speed of sound. The water vapour influence on the speed of sound is negligible and is less than 0.3%.

The graphs of the speed of sound determined by expressions [6, 7 and 8] are presented in Fig. 1. The maximum difference of the computed speed of sound is less than 2.2 m s⁻¹ and is valid for a temperature over 60°. It can be seen that the higher the temperature, the higher the speed of sound. The speed of sound is a function of the current atmosphere condition, e.g. atmospheric temperature, atmospheric humidity and atmospheric pressure.

The effect of atmospheric humidity is corrected by the using of virtual air temperature. The description of virtual temperature computation is described in [6]. Virtual temperature determination is based on the evaluation of the temperature of clear air, the density (grossness) of which is the same as the density of no clear air.

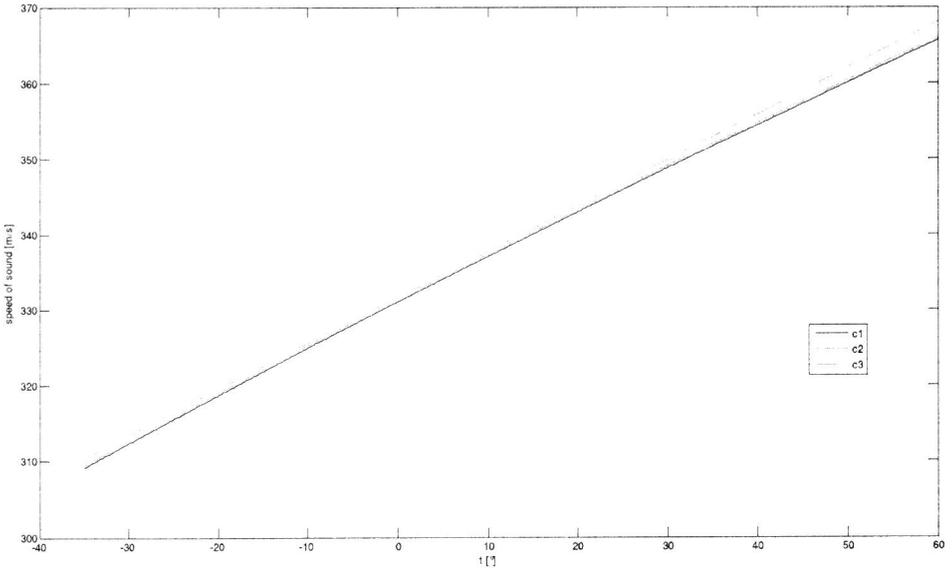


Figure 1. The speeds of sound determined by expressions [6,7,8]

The effect of speed of sound on target localization accuracy

At least three acoustic receivers (microphones) are necessary for target localization. Four microphones (moveable acoustic sensor system) in line were used in the mathematical model (see Fig. 2).

The target localization is based on a comparison of the signals from all four microphones. The distance d_{csi} of i^{th} microphone to the target is:

$$d_{csi} = vt_i = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} \quad (9)$$

where t_i is transition time of the sound wave from the target to the i^{th} microphone, v is current speed of sound, (x_c, y_c) and (x_i, y_i) are target resp. microphone coordinates.

The target location is based on computing the time difference of the arrival (TDOA) Δt_{ij} of a sound signal emitted from the target to the single receivers. The estimation of the target coordinate accuracy depends on both time TDOA measurement accuracy and local variations of the transition channel. The example of the acoustic signal records on four single receivers is presented in Fig. 3. The 1520mm self-propelled howitzer was the signal source. The single time of arrivals are marked and printed above the graph.

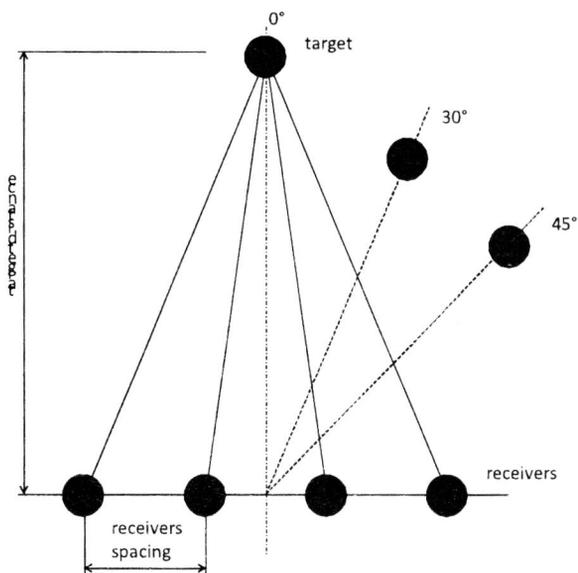


Figure 2. Target and receivers arrangement

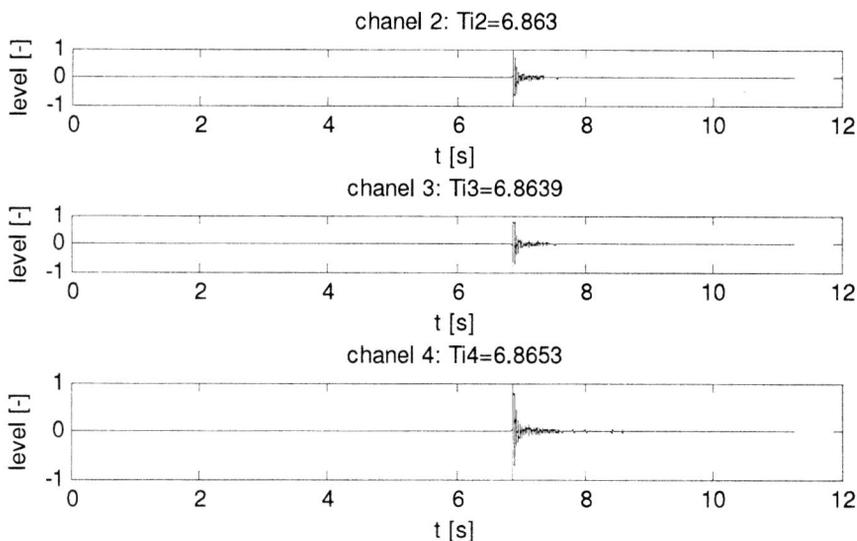


Figure 3. Acoustic signal records

The speed of sound was considered as a random variety that obeys a normal distribution. The parameters mean and variance were determined in the following way: the mean was calculated according to [9] for specific temperature. The variance was set $\sigma^2 = 1 \text{ ms}^{-1}$ which is a sufficient value (see chapter 2). A thousand executions of location computation were done for specific temperature and receivers and target arrangement. The output (resultant target location coordinates) was compared with the input (real target coordinates). Thus the population mean and population variance of the set of target location coordinates were computed (for x and y coordinate). The example of computation results can be seen in Fig.4–8.

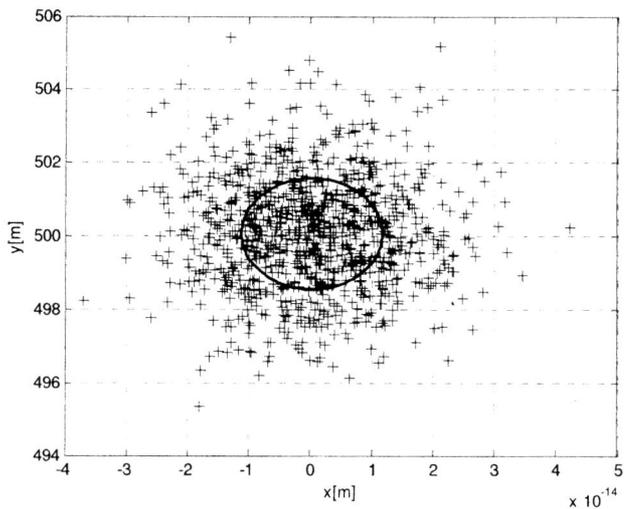


Figure 4. The position of target in distance 500m, position 0° , $\mu_x = 2.19 \cdot 10^{-16}$ [m], $\mu_y = 500,05$ [m], $\sigma_x = 1.14 \cdot 10^{-14}$ [m], $\sigma_y = 1.51$ [m] with ellipse of standard deviations

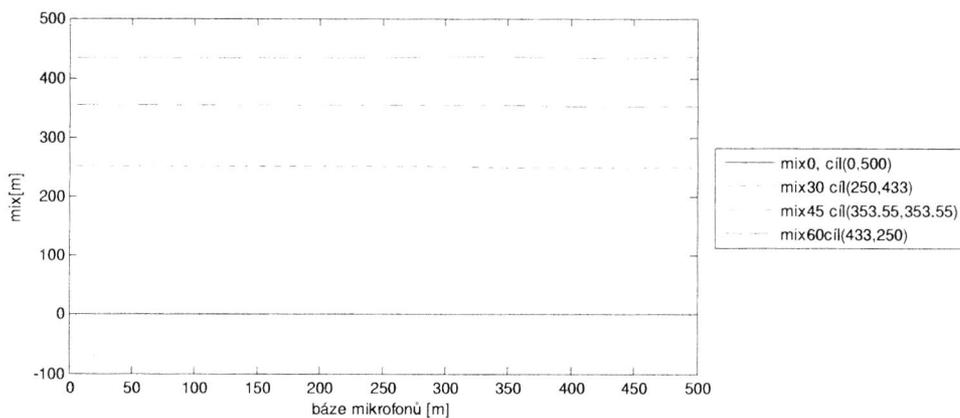


Figure 5. The mean μ_x for microphones base $b = (0.01 \text{ až } 1D)$ [m], distance $D=500$ m, target position regarding to microphones line ($0^\circ, 30^\circ, 45^\circ, 60^\circ$)

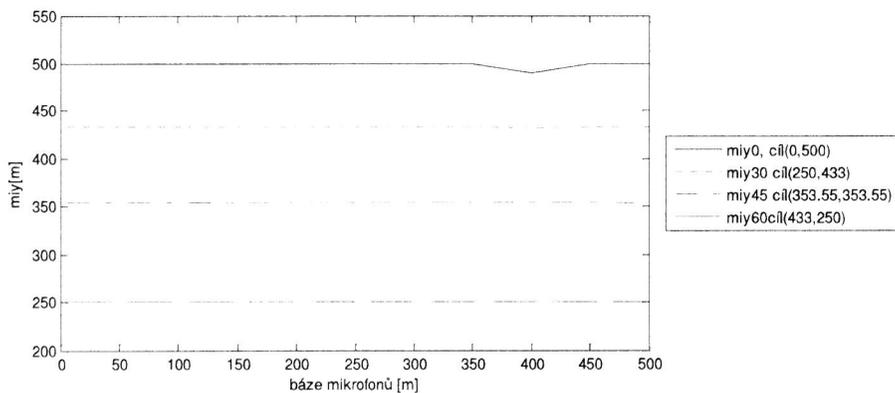


Figure 6. The mean μ_y for microphones base $b = (0.01 \text{ až } 1D)$ [m], distance $D=500$ m, target position in relation to microphone lines ($0^\circ, 30^\circ, 45^\circ, 60^\circ$)

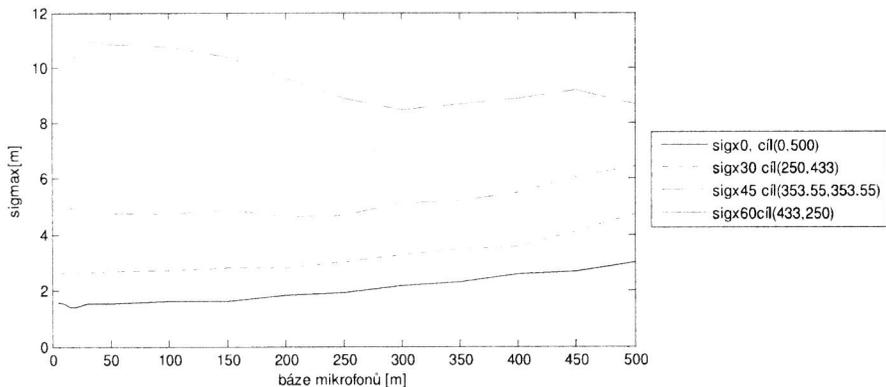


Figure 7. The standard deviations σ_x for microphones base $b = (0.01 \text{ až } 1D)$ [m], distance $D=500$ m, target position in relation to microphone lines ($0^\circ, 30^\circ, 45^\circ, 60^\circ$)

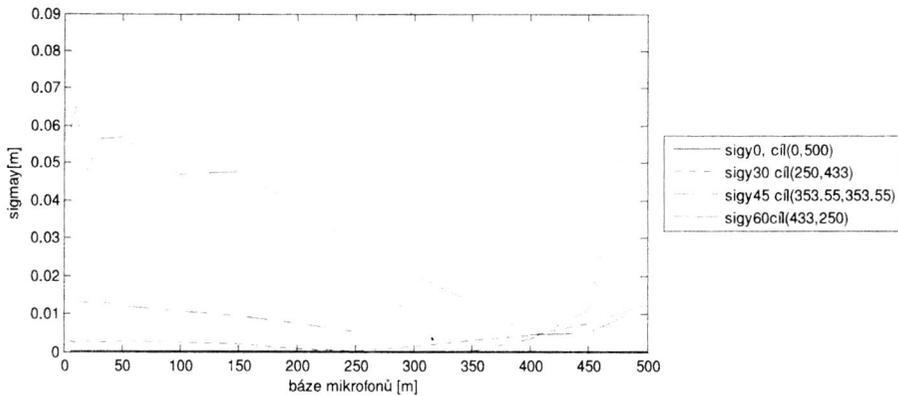


Figure 8. The standard deviations σ_y for microphones base $b = (0.01 \text{ až } 1D) [m]$, distance $D=500 \text{ m}$, target position in relation to microphone lines ($0^\circ, 30^\circ, 45^\circ, 60^\circ$)

Conclusion

Pursuant to the analysis presented here, it can be mentioned that variations to the speed of sound effects the target location minimally. An error in x-coordinate determination is significantly greater than an error in y-coordinate determination. The article was created in the conceptual Research project No.402 of Faculty of Military Technology.

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